On the Convergence Analysis of a Two-Step Modification of the Gauss-Newton Method

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Abstract

In this paper, we are concerned with finding the approximate solution of the nonlinear least squares problem [3]:

$$\min f(x) := \frac{1}{2} F(x)^T F(x), \tag{1}$$

where F is a Fréchet differentiable operator defined on R^n with its values on R^m, $m \geq n$.

For solving the problem (1), we consider a two-step modification of the Gauss-Newton method

$$\begin{cases} x_{n+1} = x_n - [F^{'}(z_n)^T F^{'}(z_n)]^{-1} F^{'}(z_n)^T F(x_n), \\ y_{n+1} = x_{n+1} - [F^{'}(z_n)^T F^{'}(z_n)]^{-1} F^{'}(z_n)^T F(x_{n+1}), \ n = 0, 1, 2, ... \end{cases} \tag{2}$$

where $z_n = (x_n + y_n)/2$; x_0 and y_0 are given. This method was initially proposed, but in a different form, by Bartish et al. [1]. The advantage of the method is that it uses only one inverse over two function evaluations.

The main focus of our study is to analyze the convergence of the method (2). The local convergence of the method (2) using the classical Lipschitz condition for second derivatives was presented in paper [2]. Instead, we study the convergence of the above-mentioned method using the Lipschitz condition with L average [4] for derivatives; such conditions are called the generalized Lipschitz conditions, where L is not a constant, but an integrable function. In addition, the radius of the convergence for the method (2) is also examined and the uniqueness ball of the solution is proved. As numerical experiments, we carry out a set of standard tests, comparing the studied method against the well-known Gauss-Newton method. Finally, the convergence analysis of the method (2) using the Lipschitz condition with L average provides the following advantages over the corresponding results in [2]: weaker convergence conditions and larger radius of the convergence.

References

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